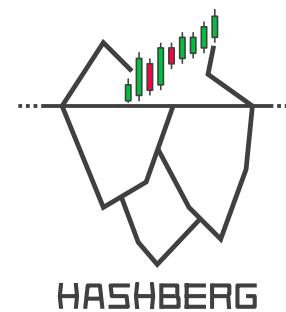


An Overview of Circuit Optimisation with the ZX Calculus

Dr. Stefano Gogioso



ZX Calculus Tensors

$$\begin{aligned} \text{---} &= |0\dots0\rangle\langle 0\dots0| + e^{i\alpha}|1\dots1\rangle\langle 1\dots1| \\ \text{---} &= |+\dots+\rangle\langle +\dots+| + e^{i\alpha}|-\dots-\rangle\langle -\dots-| \\ \text{---} &= |+\rangle\langle 0| + |-\rangle\langle 1| \end{aligned}$$

$$\begin{aligned} \text{---} &= |0\rangle\langle 0| + |1\rangle\langle 1| \\ \text{---} &= \sum_{i,j \in \{0,1\}} |ij\rangle\langle ji| \\ \text{---} &= |00\rangle + |11\rangle \\ \text{---} &= \langle 00| + \langle 11| \end{aligned}$$

ZX Calculus Rules

$$\begin{array}{c} \text{Diagram showing } \alpha + \beta \\ \text{is the sum of } \alpha \text{ and } \beta. \end{array}$$

$$\begin{array}{c} \text{---} \square \\ \text{---} \square \\ \vdots \\ \text{---} \square \end{array} \quad \text{(h)} = \quad \begin{array}{c} \text{---} \\ \text{---} \\ \vdots \\ \text{---} \\ \text{---} \end{array}$$

$$\begin{array}{c} \text{Diagram showing } \alpha \text{ connected to two red nodes} \\ = \\ \begin{array}{c} \text{Three red nodes} \\ \vdots \end{array} \end{array}$$

$$\text{---} \textcircled{1} \text{---} = \text{---}$$

(i2) _____

$$\begin{array}{c} \text{Diagram A} \\ \text{Diagram B} \end{array} = \begin{array}{c} \text{Diagram C} \\ \text{Diagram D} \end{array}$$

Quantum Circuits

$$\text{CNOT} = \begin{array}{c} \text{---} \\ | \quad | \\ \text{---} \\ | \quad | \\ \text{---} \end{array}$$

$$Z_\alpha = \text{---} \circled{\alpha} \text{---}$$

$$H = \text{---} \square \text{---}$$

$$X_\alpha = \text{---} \square \circled{\alpha} \square \text{---} = \text{---} \circled{\alpha} \text{---}$$

$$\text{CZ} = \text{---} \square \text{---} \circled{\alpha} \text{---} \square \text{---} = \text{---} \square \text{---} \circled{\alpha} \text{---} = \text{---} \circled{\alpha} \text{---}$$

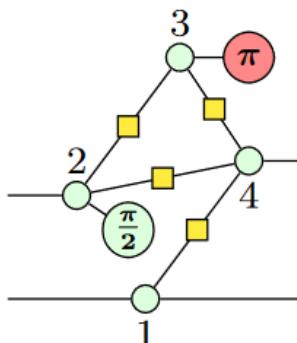
MBQC

operator	N_i	E_{ij}	$\langle +_{XY,\alpha(i)} _i$	$\langle +_{XZ,\alpha(i)} _i$	$\langle +_{YZ,\alpha(i)} _i$
diagram					

Example 2.15. The measurement pattern with the qubit register $V = \{1, 2, 3, 4\}$, input and output sets $I = \{1, 2\}$ and $O = \{1, 4\}$ and the sequence of commands

$$M_2^{XY, \frac{\pi}{2}} M_3^{YZ, \pi} E_{14} E_{23} E_{24} E_{34} N_3 N_4$$

is represented by the following ZX-diagram:



arXiv > quant-ph > arXiv:2003.01664

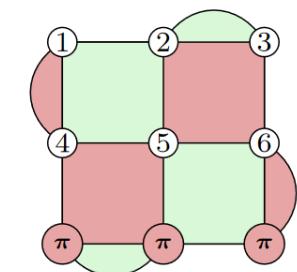
Quantum Physics

[Submitted on 3 Mar 2020 (v1), last revised 23 Mar 2021 (this version, v3)]

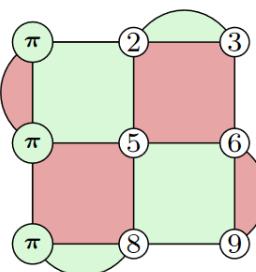
There and back again: A circuit extraction tale

Miriam Backens, Hector Miller-Bakewell, Giovanni de Felice, Leo Lobski, John van de Wetering

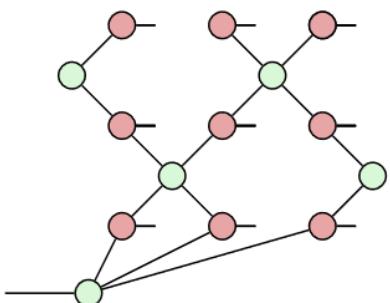
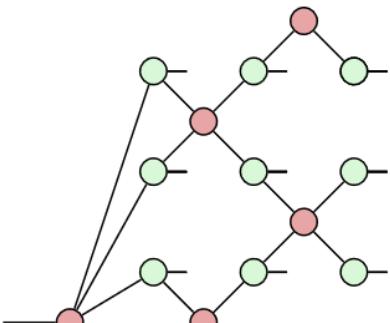
CSS Codes



$$\rightsquigarrow \overline{\mathcal{X}} := X_7 X_8 X_9$$



$$\rightsquigarrow \overline{\mathcal{Z}} := Z_1 Z_4 Z_7$$

 $=$ 

arXiv > quant-ph > arXiv:2204.14038

Quantum Physics

[Submitted on 29 Apr 2022]

Phase-free ZX diagrams are CSS codes

Aleks Kissinger

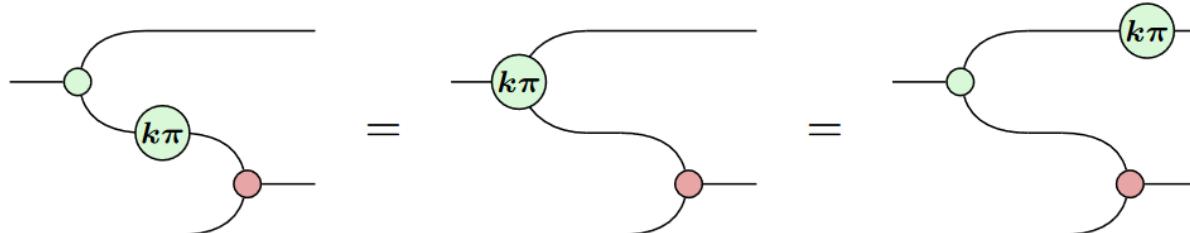
Surface Code Lattice Surgery

$$Z\text{-split} := \text{---} \circlearrowleft$$

$$X\text{-split} := \text{---} \circlearrowright$$

$$Z\text{-merge} := \left\{ \text{---} \circlearrowright \begin{array}{c} \text{---} \\[-1ex] \text{---} \end{array} \begin{array}{c} \text{---} \\[-1ex] \text{---} \end{array} \right\}_{k=0,1}$$

$$X\text{-merge} := \left\{ \text{---} \circlearrowleft \begin{array}{c} \text{---} \\[-1ex] \text{---} \end{array} \begin{array}{c} \text{---} \\[-1ex] \text{---} \end{array} \right\}_{k=0,1}$$



arXiv > quant-ph > arXiv:2204.14038

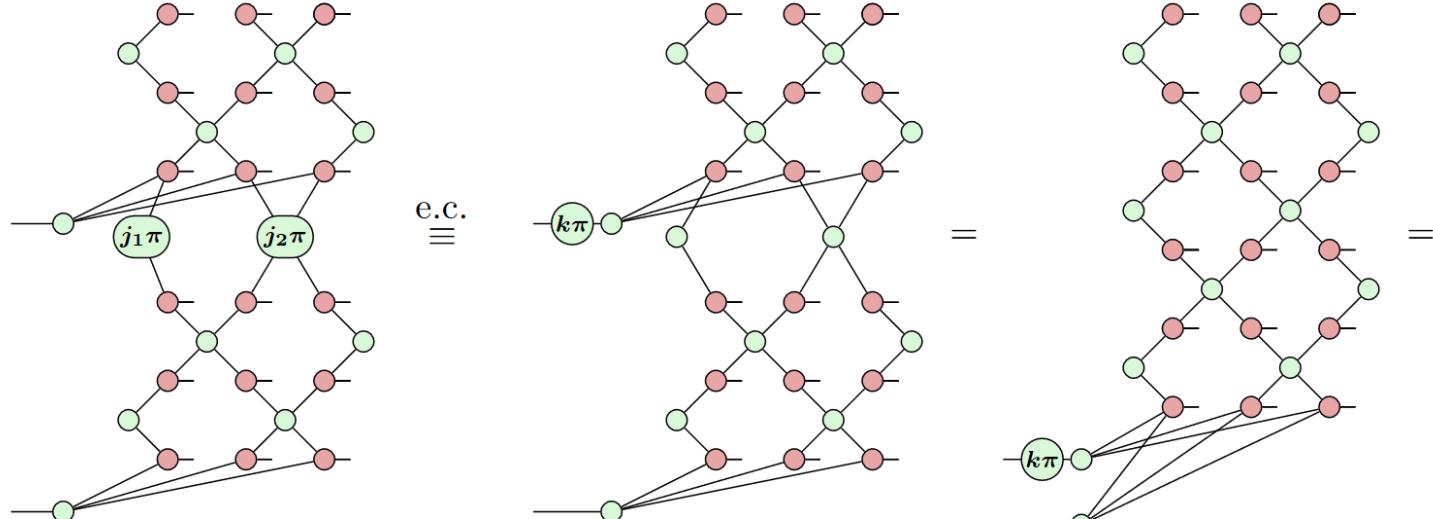
Quantum Physics

[Submitted on 29 Apr 2022]

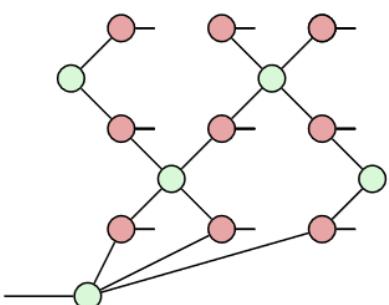
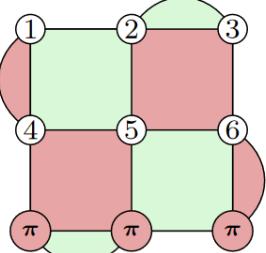
Phase-free ZX diagrams are CSS codes

Aleks Kissinger

Surface Code Lattice Surgery



$$X\text{-merge} := \left\{ \begin{array}{c} \text{---} \\ | \\ \text{---} \\ | \\ \text{---} \end{array} \right. \left. \begin{array}{c} \text{---} \\ | \\ \text{---} \\ | \\ \text{---} \end{array} \right\}_{k=0,1}$$



arXiv > quant-ph > arXiv:2204.14038

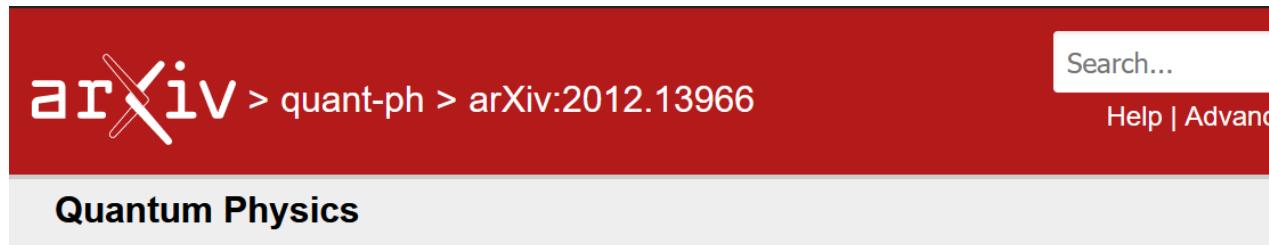
Quantum Physics

[Submitted on 29 Apr 2022]

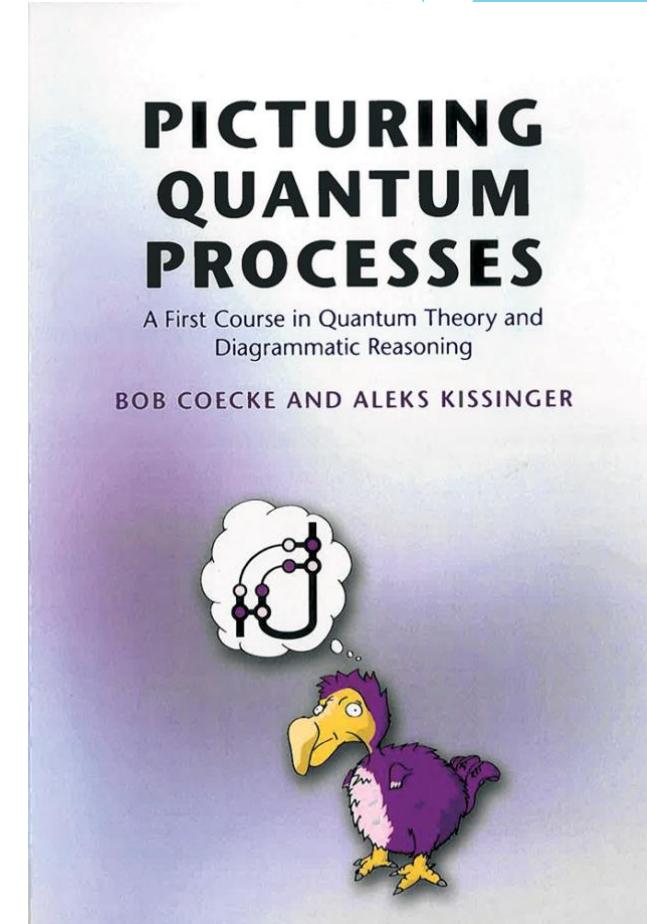
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Aleks Kissinger

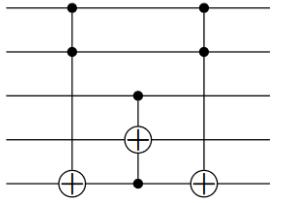
References



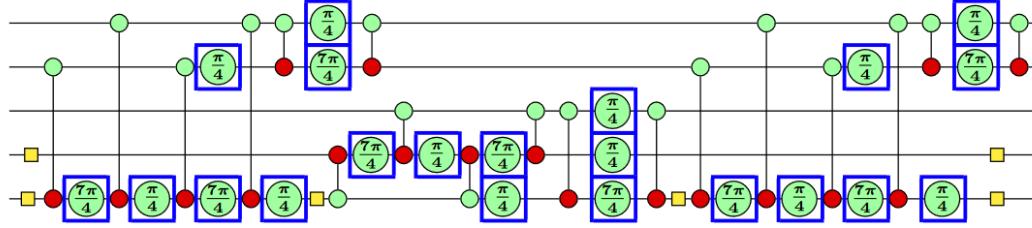
The image shows a screenshot of the arXiv search interface. At the top, there is a red header bar with the arXiv logo and navigation links for "quant-ph" and "arXiv:2012.13966". To the right of the header are search and help functions. Below the header, a grey bar contains the text "Quantum Physics". The main content area features a submission by "John van de Wetering" titled "ZX-calculus for the working quantum computer scientist", submitted on "27 Dec 2020".



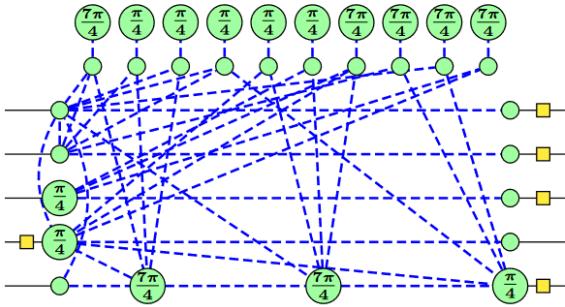
T-Count Reduction



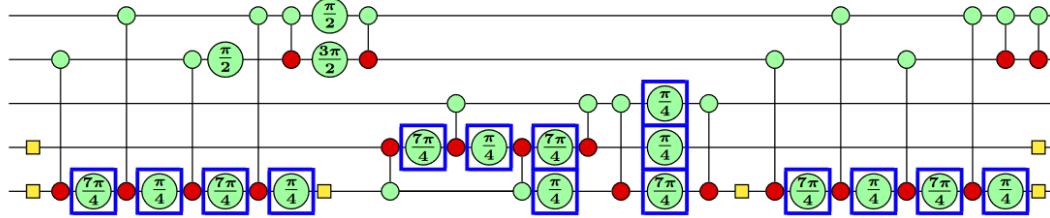
(a) Original circuit



(b) The circuit expanded as a ZX-diagram, with 21 T gates.



(c) Simplified ZX-diagram.



(d) 15 T gates remain after phase-teleportation.

arXiv > quant-ph > arXiv:1904.04735

Quantum Physics

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PyZX: Large Scale Automated Diagrammatic Reasoning

Aleks Kissinger, John van de Wetering

arXiv > quant-ph > arXiv:1903.10477

Quantum Physics

[Submitted on 25 Mar 2019 (v1), last revised 17 Jan 2020 (this version, v3)]

Reducing T-count with the ZX-calculus

Aleks Kissinger, John van de Wetering

$$\dots \begin{array}{c} \alpha \\ \diagup \quad \diagdown \\ \text{---} \end{array} \begin{array}{c} \beta \\ \diagup \quad \diagdown \\ \text{---} \end{array} \dots = \dots \begin{array}{c} \alpha \\ \diagup \quad \diagdown \\ \text{---} \end{array} \begin{array}{c} \beta \\ \diagup \quad \diagdown \\ \text{---} \end{array} \dots$$

$$\dots \begin{array}{c} \alpha \\ \diagup \quad \diagdown \\ \text{---} \end{array} = \dots \begin{array}{c} \alpha \\ \diagup \quad \diagdown \\ \text{---} \end{array}$$

$$\dots \begin{array}{c} \alpha \\ \diagup \quad \diagdown \\ \text{---} \end{array} = \dots \begin{array}{c} \alpha \\ \diagup \quad \diagdown \\ \text{---} \end{array}$$

$$\dots \begin{array}{c} \alpha \\ \diagup \quad \diagdown \\ \text{---} \end{array} = \dots \begin{array}{c} \alpha + \pi \\ \diagup \quad \diagdown \\ \text{---} \end{array}$$

$$\begin{array}{c} \alpha_1 \quad \dots \quad \alpha_n \\ \diagup \quad \dots \quad \diagdown \\ \text{---} \end{array} \begin{array}{c} \pm \frac{\pi}{2} \\ \diagup \quad \diagdown \\ \text{---} \end{array} = \begin{array}{c} \alpha_1 \mp \frac{\pi}{2} \\ \diagup \quad \dots \quad \diagdown \\ \text{---} \end{array} \dots \begin{array}{c} \alpha_n \mp \frac{\pi}{2} \\ \diagup \quad \dots \quad \diagdown \\ \text{---} \end{array}$$

$$(LC) = \begin{array}{c} \alpha_1 \mp \frac{\pi}{2} \quad \dots \quad \alpha_n \mp \frac{\pi}{2} \\ \diagup \quad \dots \quad \diagdown \\ \text{---} \end{array}$$

$$\begin{array}{c} \dots \quad u \quad \dots \quad v \quad \dots \\ \diagup \quad \diagdown \quad \diagup \quad \diagdown \\ \text{---} \quad \text{---} \quad \text{---} \quad \text{---} \\ \alpha_1 \quad \dots \quad \beta_n \quad \dots \quad \gamma_n \\ \diagup \quad \dots \quad \diagdown \quad \dots \quad \diagdown \\ \text{---} \quad \text{---} \quad \text{---} \quad \text{---} \\ \alpha_n \quad \dots \quad \beta_1 \quad \dots \quad \gamma_1 \\ \diagdown \quad \dots \quad \diagup \quad \dots \quad \diagup \\ \text{---} \quad \text{---} \quad \text{---} \quad \text{---} \\ U \qquad \qquad \qquad V \qquad \qquad \qquad W \end{array}$$

$$(P1) = \begin{array}{c} \dots \quad \alpha_1 + k\pi \quad \dots \quad \gamma_1 + j\pi \quad \dots \\ \diagup \quad \diagdown \quad \diagup \quad \diagdown \\ \text{---} \quad \text{---} \quad \text{---} \quad \text{---} \\ \alpha_n + k\pi \quad \dots \quad \gamma_n + j\pi \quad \dots \\ \diagup \quad \diagdown \quad \diagup \quad \diagdown \\ \text{---} \quad \text{---} \quad \text{---} \quad \text{---} \\ \beta_n + (j+k+1)\pi \\ \dots \\ \beta_1 + (j+k+1)\pi \quad \dots \end{array}$$

$$\begin{array}{c} \dots \quad \alpha_1 \quad \dots \\ \diagup \quad \diagdown \quad \dots \\ \beta \quad \alpha \quad \dots \\ \diagup \quad \diagdown \\ \text{---} \end{array} (f) = \begin{array}{c} \dots \quad \alpha_1 \quad \dots \\ \diagup \quad \diagdown \quad \dots \\ \beta \quad \alpha \quad \dots \\ \diagup \quad \diagdown \\ \text{---} \end{array} (h) = \begin{array}{c} \dots \quad \alpha_1 \quad \dots \\ \diagup \quad \diagdown \quad \dots \\ \beta \quad \alpha \quad \dots \\ \diagup \quad \diagdown \\ \text{---} \end{array}$$

$$= \begin{array}{c} \dots \quad \alpha_1 \quad \dots \\ \diagup \quad \diagdown \quad \dots \\ \beta \quad \alpha \quad \dots \\ \diagup \quad \diagdown \\ \text{---} \end{array} (b') = \begin{array}{c} \dots \quad \alpha_1 \quad \dots \\ \diagup \quad \diagdown \quad \dots \\ \beta \quad \alpha \quad \dots \\ \diagup \quad \diagdown \\ \text{---} \end{array} (f) = \begin{array}{c} \dots \quad \alpha_1 \quad \dots \\ \diagup \quad \diagdown \quad \dots \\ \alpha + \beta \quad \alpha_1 \quad \dots \\ \diagup \quad \diagdown \\ \text{---} \end{array}$$

$$(h) = \begin{array}{c} \dots \quad \alpha_1 \quad \dots \\ \diagup \quad \diagdown \quad \dots \\ \alpha + \beta \quad \alpha_1 \quad \dots \\ \diagup \quad \diagdown \\ \text{---} \end{array}$$

arXiv > quant-ph > arXiv:1904.04735

Quantum Physics

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PyZX: Large Scale Automated Diagrammatic Reasoning

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Quantum Physics

[Submitted on 25 Mar 2019 (v1), last revised 17 Jan 2020 (this version, v3)]

Reducing T-count with the ZX-calculus

Aleks Kissinger, John van de Wetering

Circuit	<i>n</i>	T	Best prev.	Method	PyZX	PyZX+TODD
adder ₈ [3]	24	399	213	RM _m	173	167
Adder8 [27]	23	266	56	NRSCM	56	56
Adder16 [27]	47	602	120	NRSCM	120	120
Adder32 [27]	95	1274	248	NRSCM	248	248
Adder64 [27]	191	2618	504	NRSCM	504	504
barenco-tof3 [27]	5	28	16	Tpar	16	16
barenco-tof4 [27]	7	56	28	Tpar	28	28
barenco-tof5 [27]	9	84	40	Tpar	40	40
barenco-tof10 [27]	19	224	100	Tpar	100	100
tof ₃ [27]	5	21	15	Tpar	15	15
tof ₄ [27]	7	35	23	Tpar	23	23
tof ₅ [27]	9	49	31	Tpar	31	31
tof ₁₀ [27]	19	119	71	Tpar	71	71
csla-mux ₃ [27]	15	70	58	RM _r	62	45
csum-mux ₉ [27]	30	196	76	RM _r	84	72
cycle17 ₃ [3]	35	4739	1944	RM _m	1797	1797
gf(2 ⁴)-mult [27]	12	112	56	TODD	68	52
gf(2 ⁵)-mult [27]	15	175	90	TODD	115	86
gf(2 ⁶)-mult [27]	18	252	132	TODD	150	122
gf(2 ⁷)-mult [27]	21	343	185	TODD	217	173
gf(2 ⁸)-mult [27]	24	448	216	TODD	264	214
ham15-low [3]	17	161	97	Tpar	97	97
ham15-med [3]	17	574	230	Tpar	212	212
ham15-high [3]	20	2457	1019	Tpar	1019	1013
hwb ₆ [3]	7	105	75	Tpar	75	72
hwb ₈ [3]	12	5887	3531	RM _{m&r}	3517	3501
mod-mult-55 [27]	9	49	28	TODD	35	20
mod-red-21 [27]	11	119	73	Tpar	73	73
mod5 ₄ [27]	5	28	16	Tpar	8	7
nth-prime ₆ [3]	9	567	400	RM _{m&r}	279	279
nth-prime ₈ [3]	12	6671	4045	RM _{m&r}	4047	3958
qcla-adder ₁₀ [27]	36	589	162	Tpar	162	158
qcla-com ₇ [27]	24	203	94	RM _m	95	91
qcla-mod ₇ [27]	26	413	235 ^a	NRSCM	237	216
rc-adder ₆ [27]	14	77	47	RM _{m&r}	47	47
vbe-adder ₃ [27]	10	70	24	Tpar	24	24

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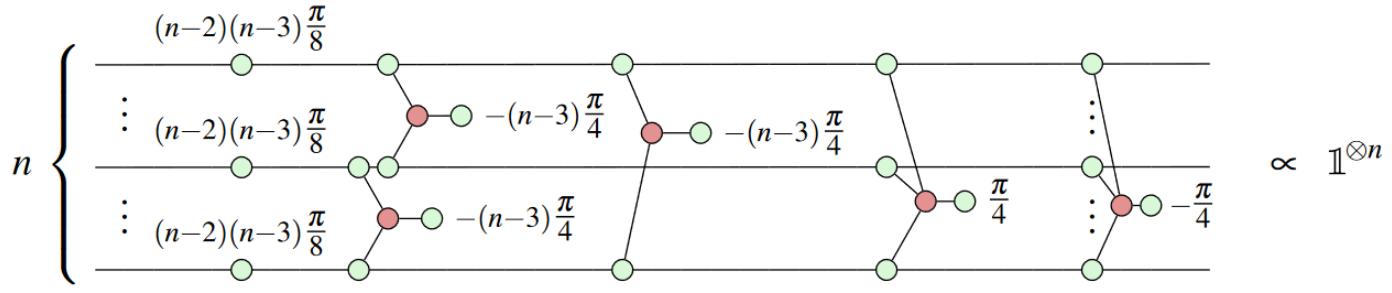
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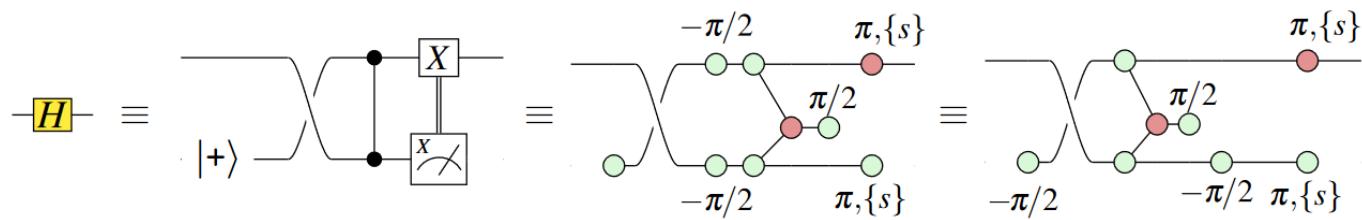
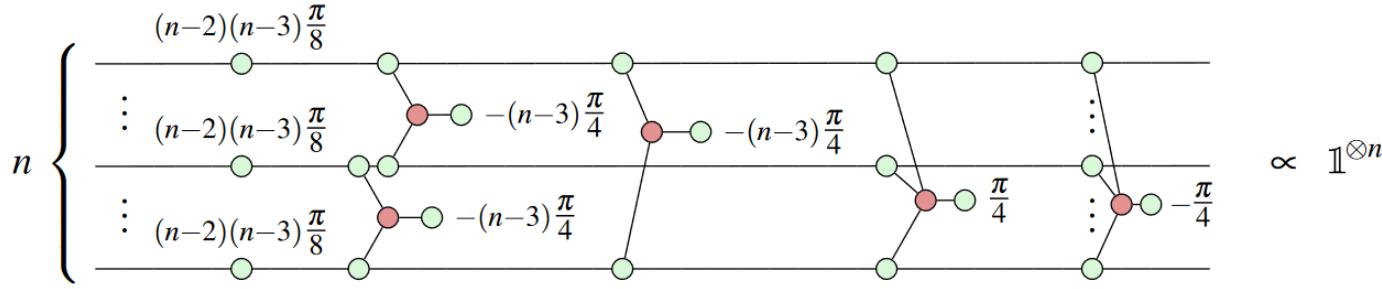
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Quantum Physics

[Submitted on 20 Nov 2019 (v1), last revised 1 May 2020 (this version, v2)]

Techniques to Reduce $\pi/4$ -Parity-Phase Circuits, Motivated by the ZX Calculus

Niel de Beaudrap (Department of Computer Science, University of Oxford),
Xiaoning Bian (Department of Mathematics and Statistics, Dalhousie
University), Quanlong Wang (Department of Computer Science, University of
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arXiv > quant-ph > arXiv:1911.09039v2

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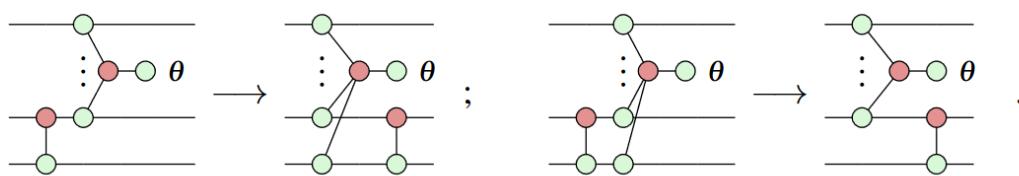
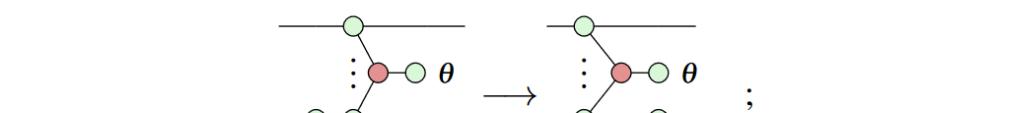
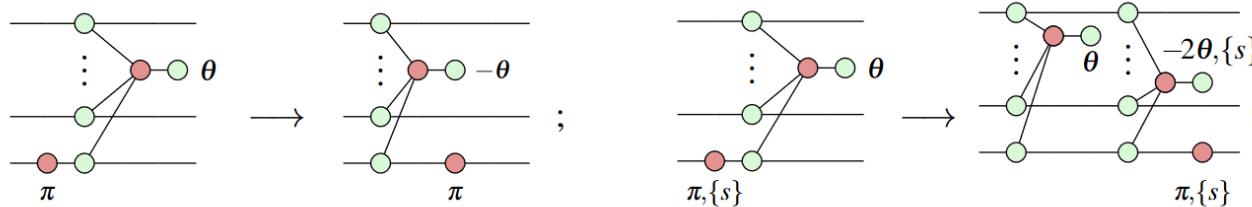
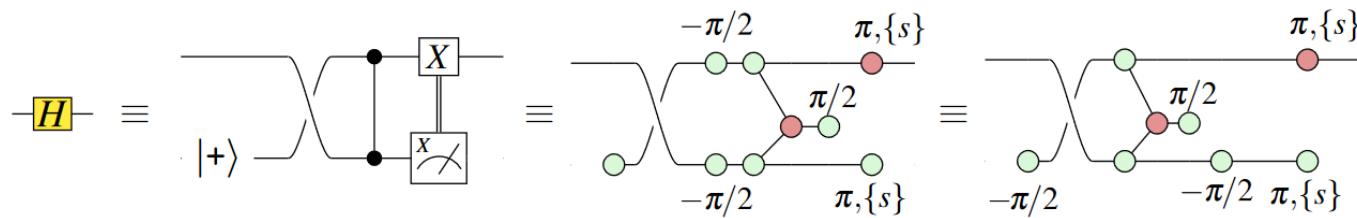
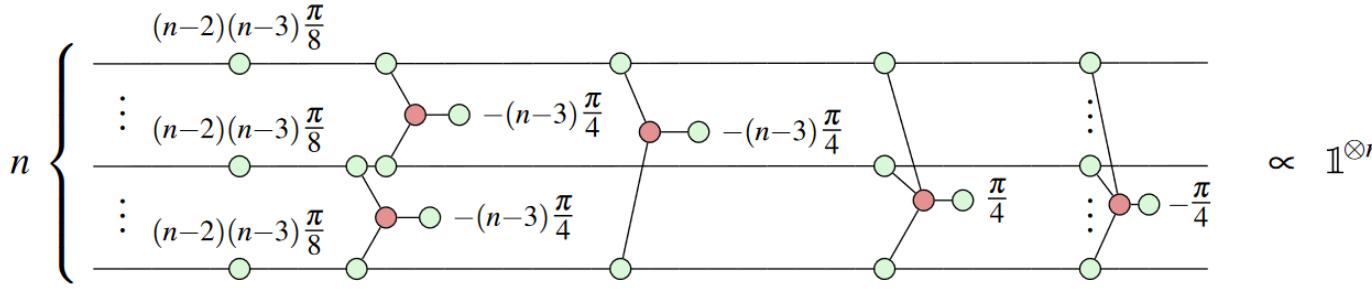
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$\propto \mathbb{I}^{\otimes n}$

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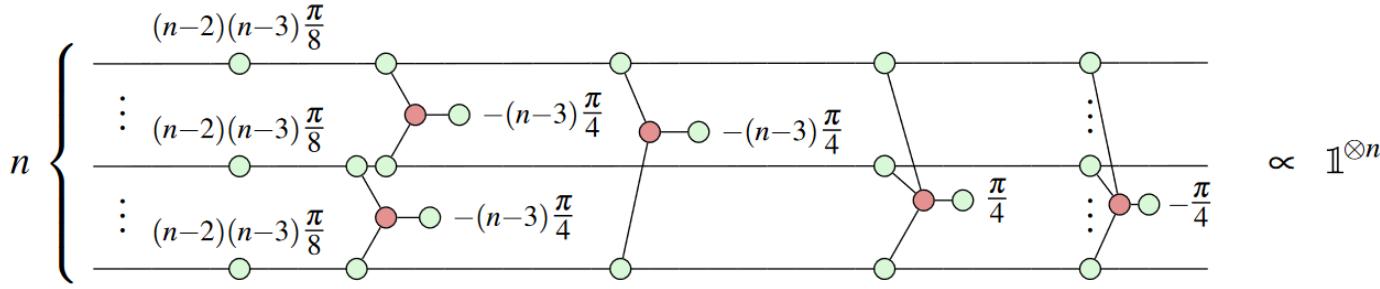
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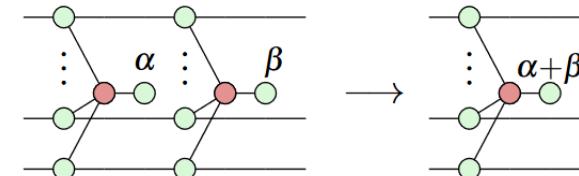
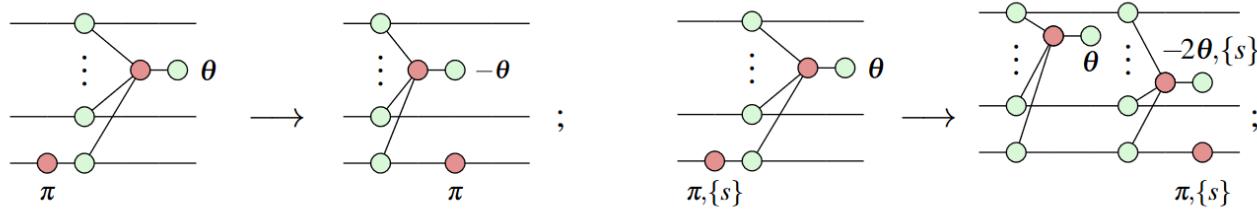
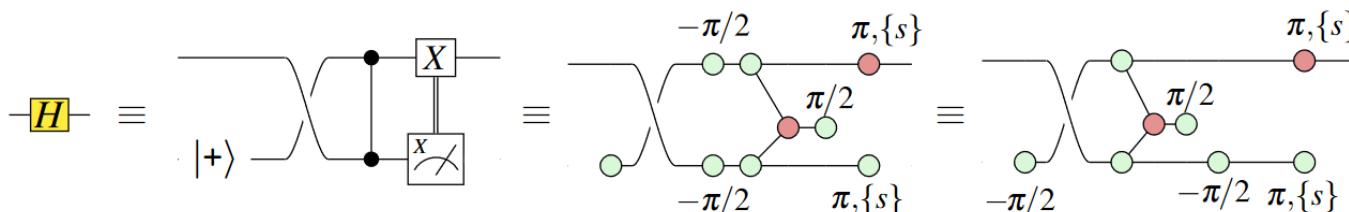
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$\propto \mathbb{I}^{\otimes n}$



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Circuit	# qubits			T count & optimisation						
	n input	δn [23]	δn (ours)	init. #T	final #T (prev. opt.)	Ref.	time (s)	final #T (our results)	time (s)	Verified? (feynver)
Adder ₈	24	71	41	399	212 ^(a) [23]	227.81		176*	24.62	✓
Barenco Tof ₃	5	3	3	28	14 ^(b) [23]	0.01*		13*	0.07607	✓
Barenco Tof ₄	7	7	7	56	24 [23]	0.45		25	1.884	✓
Barenco Tof ₅	9	11	11	84	34 [23]	1.94		37	13.76	✓
Barenco Tof ₁₀	19	31	31	224	84 [23]	460.33		97	24.49	✓
CSLA MUX ₃	15	17	6	70	40 ^(b) [23]	3.73		44	18.01	✓
CSUM MUX ₉	30	12	12	196	74 ^(a) [23]	36.57		84	23.98	✓
GF(2 ⁴) Mult	12	7	0	112	56 ^(b) [23]	0.55		53*	0.8180	✓
GF(2 ⁵) Mult	15	9	0	175	90 ^(b) [23]	6.96		88*	4.279	✓
GF(2 ⁶) Mult	18	11	0	252	132 ^(b) [23]	121.16		128*	7.894	✓
GF(2 ⁷) Mult	21	13	0	343	185 ^(a) [23]	153.75		167*	27.21	✓
GF(2 ⁸) Mult	24	15	0	448	216 ^(a) [23]	517.63		229	95.63	✓
GF(2 ⁹) Mult	27	17	0	567	295 [23]	3213.53		306	24.79	✓
GF(2 ¹⁰) Mult	30	19	0	700	351 [23]	23969.1		357	24.65	✓
GF(2 ¹⁶) Mult	48	31	0	1 792	922 [23]	76312.5		972	25.65	✓ ^(d)
GF(2 ³²) Mult	96	—	0	7 168	4 128 [31]	1.834		3 936*	26.07	✓ ^(d)
GF(2 ⁶⁴) Mult	192	—	0	28 672	16 448 [31]	58.341		15 865*	29.73	—
GF(2 ¹²⁸) Mult	384	—	0	114 688	65 664 [31]	1744.746		64 461*	48.78	—
GF(2 ²³¹) Mult	393	—	0	120 127	69 037 [31]	1953.353		67 772*	53.39	—
GF(2 ¹⁶³) Mult	489	—	0	185 983	106 765 [31]	4955.927		105 182*	66.27	—
GF(2 ²⁵⁶) Mult	768	—	0	458 752	—	—		260 539*	137.1	—
GF(2 ⁵¹²) Mult	1536	—	0	1 835 008	—	—		1 046 964*	850.7 ^(d)	—
Mod5 ₄	5	6	0	28	16 ^(b) [31]	0.001*		7*	0.00899	✓
Mod Adder ₁₀₂₄	28	$\leq 270^{(c)}$	304	1 995	978 [23]	665.5		1 010	27.56	✓ ^(d)
Mod Mult ₅₅	9	10	3	49	28 ^(a) [23]	0.02		19*	0.5775	✓
Mod Red ₂₁	11	17	17	119	69 ^(b) [23]	0.59		65	27.68	✓
QCLA Adder ₁₀	36	28	25	238	157 [23]	366.1		147*	24.96	✓
QCLA Com ₇	24	19	18	203	81 [23]	170.77		84	24.21	✓
QCLA Mod ₇	26	58	58	413	221 ^(a) [23]	289.77		233	24.26	✓ ^(d)
RC Adder ₆	14	21	10	77	45 ^(b) [23]	0.97		38	30.70	✓
NC Toff ₃	5	2	2	21	13 [23]	0.01*		13*	0.04005	✓
NC Toff ₄	7	4	4	35	19 [23]	0.06		19*	0.5322	✓
NC Toff ₅	9	11	6	49	25 [23]	0.4		26	2.910	✓
NC Toff ₁₀	19	16	16	119	55 [23]	44.78		60	28.01	✓
VBE Adder ₃	10	4	4	70	20 [23]	0.15		20*	1.896	✓

arXiv > quant-ph > arXiv:1911.09039v2

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Quantum Physics

[Submitted on 20 Nov 2019 (v1), last revised 1 May 2020 (this version, v2)]

Techniques to Reduce $\pi/4$ -Parity-Phase Circuits, Motivated by the ZX Calculus

Niel de Beaudrap (Department of Computer Science, University of Oxford), Xiaoning Bian (Department of Mathematics and Statistics, Dalhousie University), Quanlong Wang (Department of Computer Science, University of Oxford, Cambridge Quantum Computing Ltd.)

arXiv > quant-ph > arXiv:2004.05164

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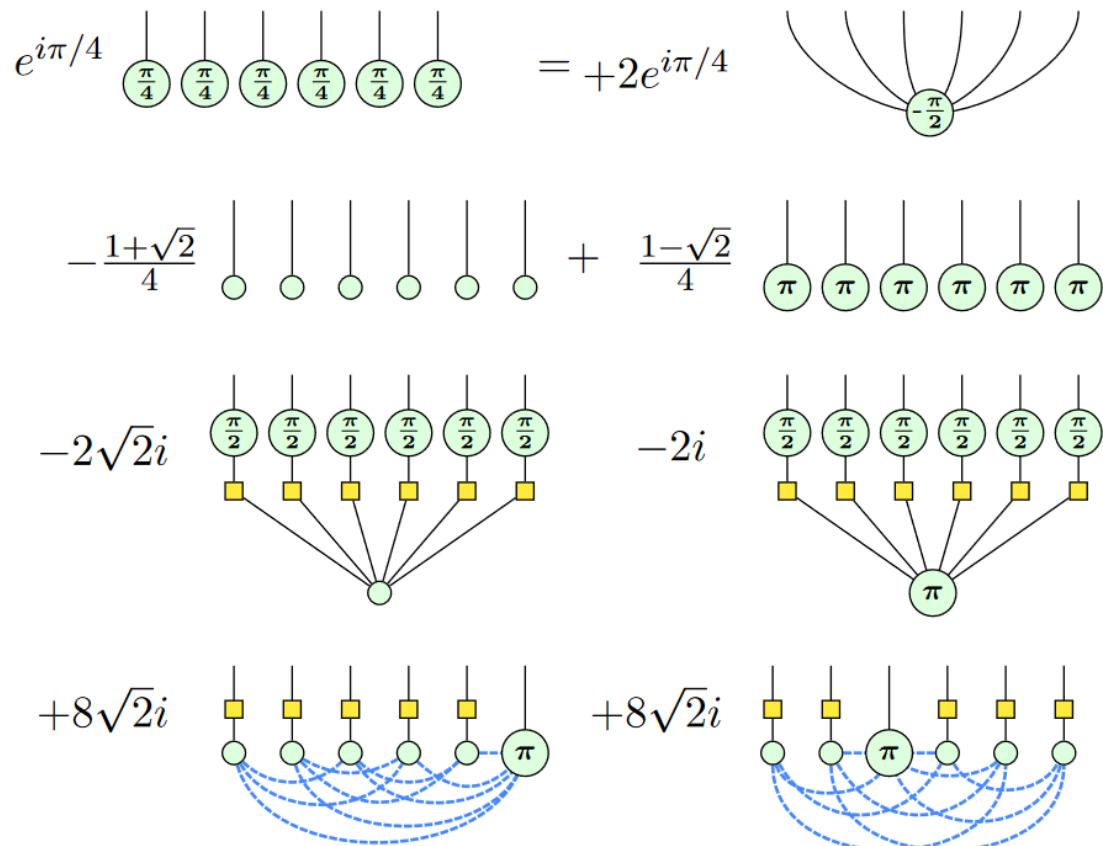
Quantum Physics

[Submitted on 10 Apr 2020 (v1), last revised 14 Apr 2020 (this version, v2)]

Fast and effective techniques for T-count reduction via spider nest identities

Niel de Beaudrap, Xiaoning Bian, Quanlong Wang

Simulation Techniques



arXiv > quant-ph > arXiv:2109.01076

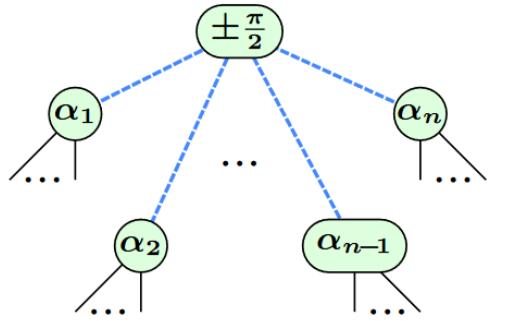
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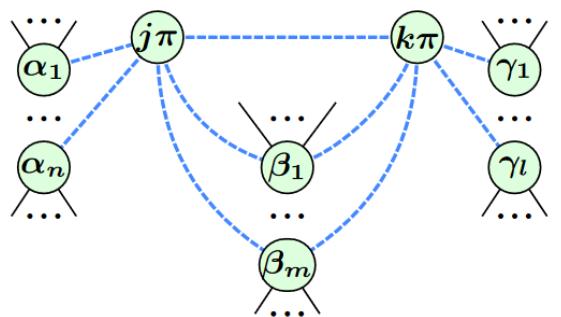
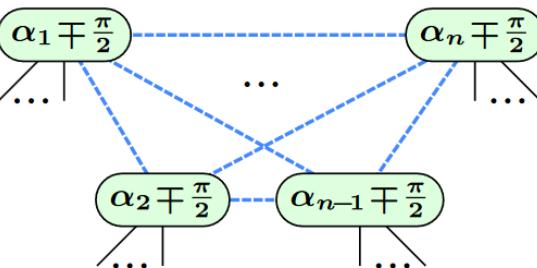
[Submitted on 2 Sep 2021]

Simulating quantum circuits with ZX-calculus reduced stabiliser decompositions

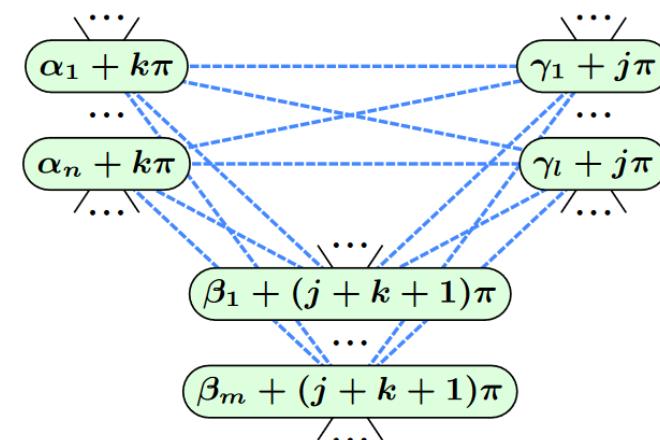
Aleks Kissinger, John van de Wetering



$$= e^{\pm i\pi/4} \sqrt{2}^{\frac{(n-1)(n-2)}{2}}$$



$$= (-1)^{jk} \sqrt{2}^E$$



$$\dots \xrightarrow{\alpha} \xrightarrow{\beta} \dots = \frac{1}{2} \dots \xrightarrow{\alpha} \xrightarrow{\beta} \dots$$

$$\dots \xrightarrow{\alpha} \dots = \xrightarrow{\alpha} \dots$$

$$\dots \xrightarrow{\alpha} \xrightarrow{\text{dashed loop}} \dots = \frac{1}{\sqrt{2}} \xrightarrow{\alpha + \pi} \dots$$

arXiv > quant-ph > arXiv:2109.01076

Quantum Physics

[Submitted on 2 Sep 2021]

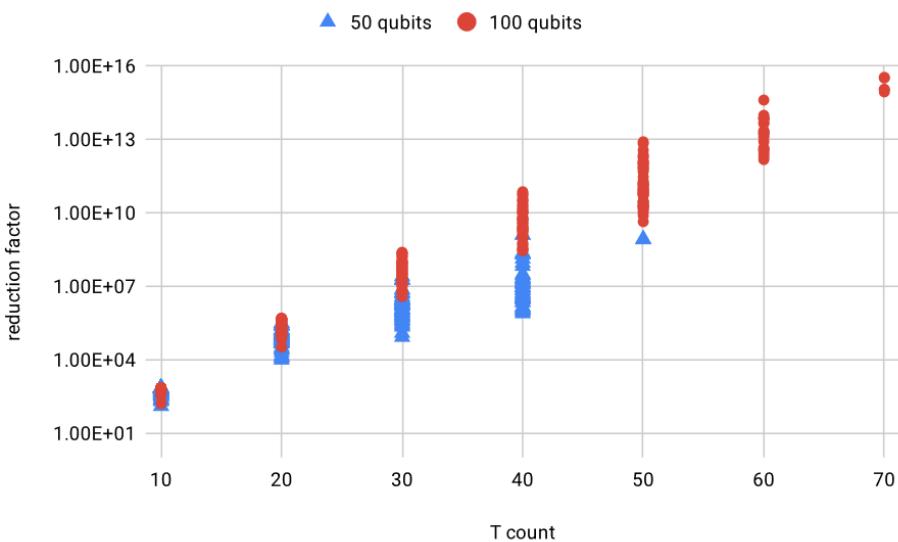
Simulating quantum circuits with ZX-calculus reduced stabiliser decompositions

Aleks Kissinger, John van de Wetering

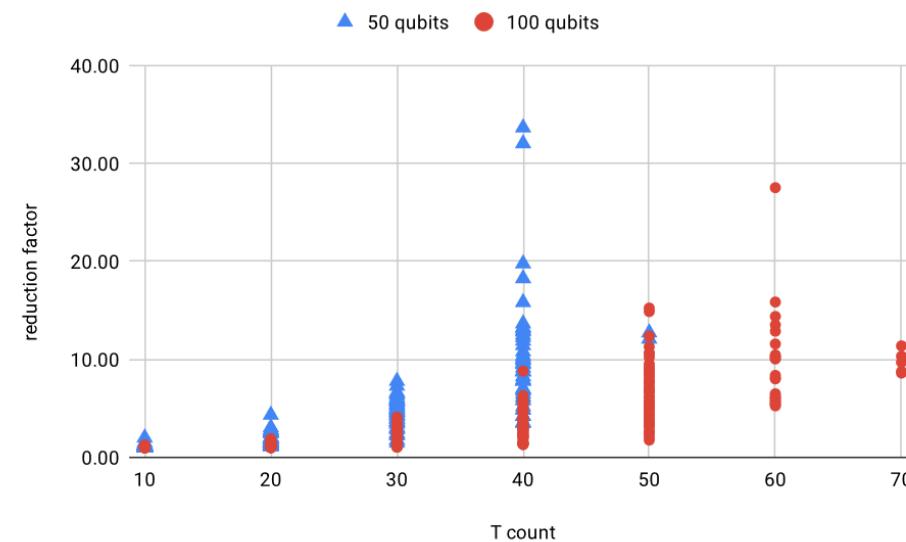
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Reduction vs. naïve BSS decomposition



Reduction vs. simplified BSS decomposition



arXiv > quant-ph > arXiv:2109.01076

Quantum Physics

[Submitted on 2 Sep 2021]

Simulating quantum circuits with ZX-calculus reduced stabiliser decompositions

Aleks Kissinger, John van de Wetering

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$$|\text{cat}_n\rangle = \frac{1}{\sqrt{2}} \left(|\psi_1\rangle - |\psi_2\rangle - \dots - |\psi_n\rangle \right)$$

 arXiv > quant-ph > arXiv:2202.09202

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Quantum Physics

[Submitted on 18 Feb 2022]

Classical simulation of quantum circuits with partial and graphical stabiliser decompositions

Aleks Kissinger, John van de Wetering, Renaud Vilmar

$$|\text{cat}_n\rangle = \frac{1}{\sqrt{2}} \circlearrowleft \begin{array}{c} \frac{\pi}{4} \\ \frac{\pi}{4} \\ \vdots \\ \frac{\pi}{4} \end{array}$$

$$\circlearrowleft \begin{array}{c} \frac{\pi}{4} \\ \frac{\pi}{4} \\ \frac{\pi}{4} \\ \frac{\pi}{4} \\ \frac{\pi}{4} \\ \frac{\pi}{4} \end{array} = \frac{1}{2} \circlearrowleft \begin{array}{c} -\frac{\pi}{2} \end{array} + \frac{ie^{i\pi/4}}{\sqrt{2}} \circlearrowleft \begin{array}{c} \frac{\pi}{4} \\ \frac{\pi}{4} \\ \frac{\pi}{4} \\ \frac{\pi}{4} \end{array} - \frac{e^{i\pi/4}}{\sqrt{2}} \circlearrowleft \begin{array}{c} \frac{\pi}{2} \\ \frac{\pi}{2} \\ \frac{\pi}{2} \\ \frac{\pi}{2} \end{array}$$

$$\circlearrowleft \begin{array}{c} \frac{\pi}{4} \\ \frac{\pi}{4} \\ \frac{\pi}{4} \\ \frac{\pi}{4} \end{array} = \frac{e^{-i\pi/4}}{\sqrt{2}} \circlearrowleft \begin{array}{c} -\frac{\pi}{2} \end{array} + i \circlearrowleft \begin{array}{c} \frac{\pi}{4} \\ \frac{\pi}{4} \\ \frac{\pi}{4} \end{array}$$

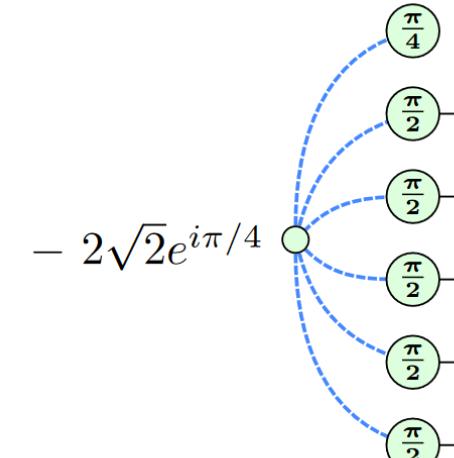
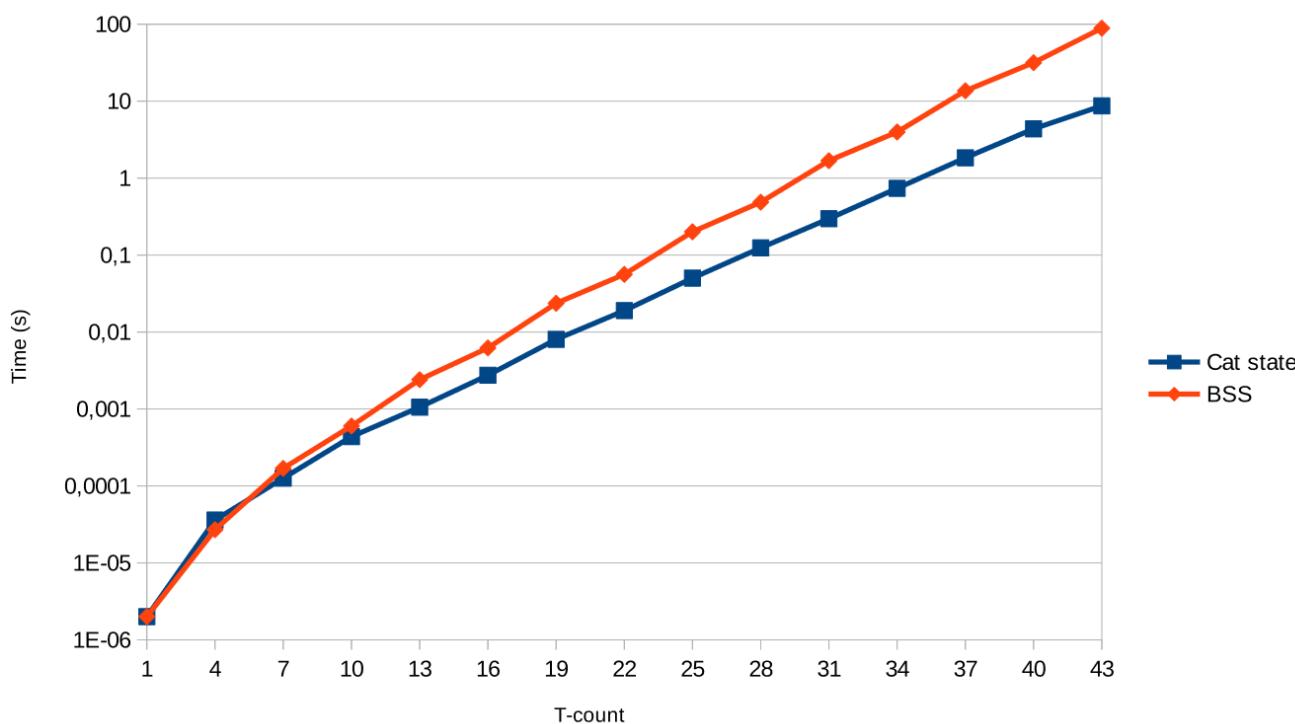
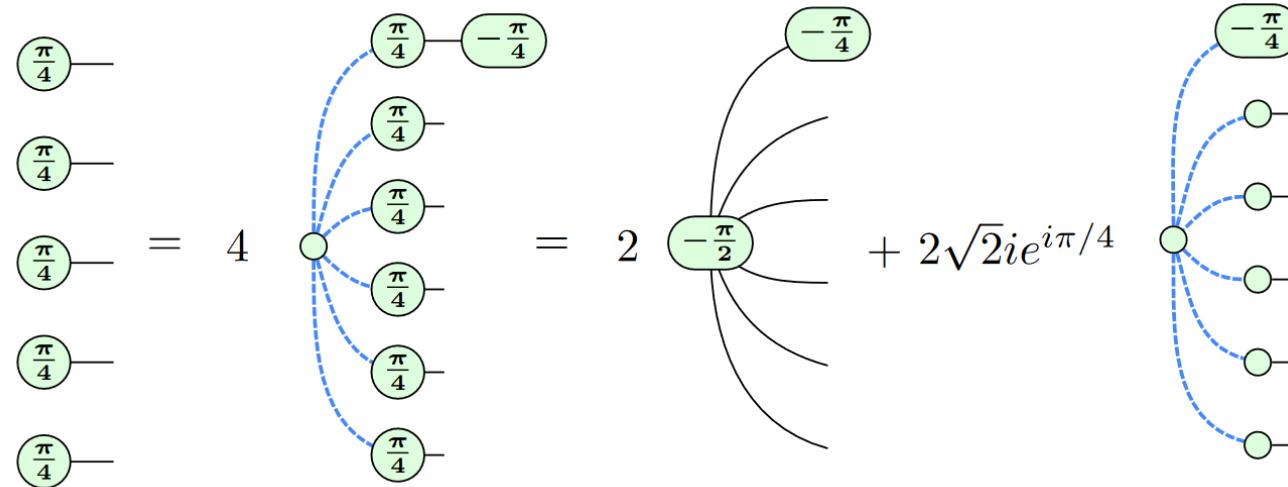
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Quantum Physics

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Aleks Kissinger, John van de Wetering, Renaud Vilmar



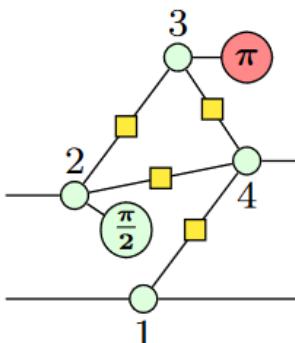
Circuit Extraction Techniques

operator	N_i	E_{ij}	$\langle +_{XY,\alpha(i)} _i$	$\langle +_{XZ,\alpha(i)} _i$	$\langle +_{YZ,\alpha(i)} _i$
diagram					

Example 2.15. The measurement pattern with the qubit register $V = \{1, 2, 3, 4\}$, input and output sets $I = \{1, 2\}$ and $O = \{1, 4\}$ and the sequence of commands

$$M_2^{XY, \frac{\pi}{2}} M_3^{YZ, \pi} E_{14} E_{23} E_{24} E_{34} N_3 N_4$$

is represented by the following ZX-diagram:



arXiv > quant-ph > arXiv:2003.01664

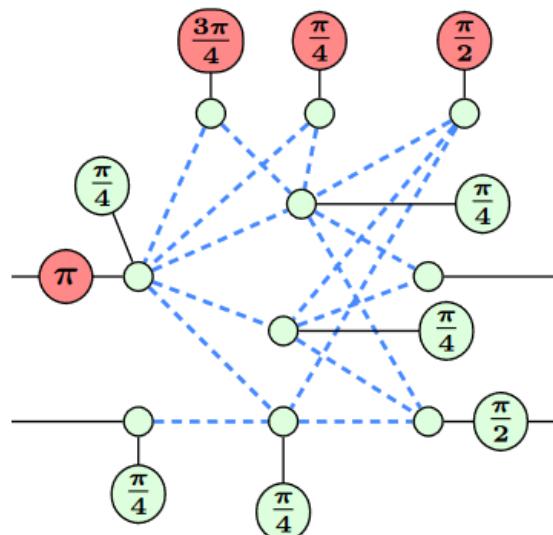
Quantum Physics

[Submitted on 3 Mar 2020 (v1), last revised 23 Mar 2021 (this version, v3)]

There and back again: A circuit extraction tale

Miriam Backens, Hector Miller-Bakewell, Giovanni de Felice, Leo Lobski, John van de Wetering

operator	N_i	E_{ij}	$\langle +_{XY,\alpha(i)} _i$	$\langle +_{XZ,\alpha(i)} _i$	$\langle +_{YZ,\alpha(i)} _i$
diagram					



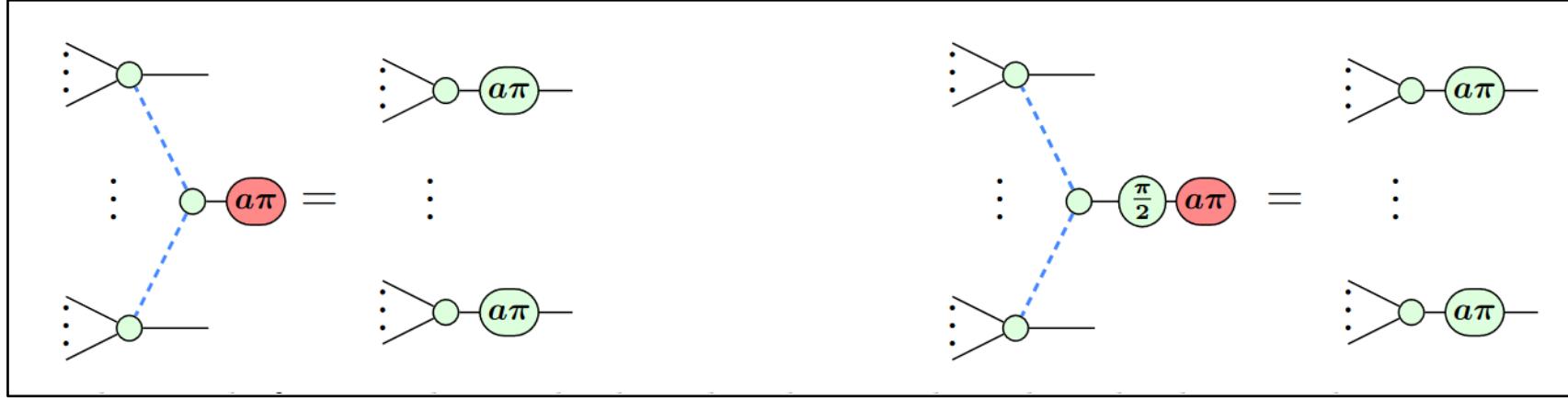
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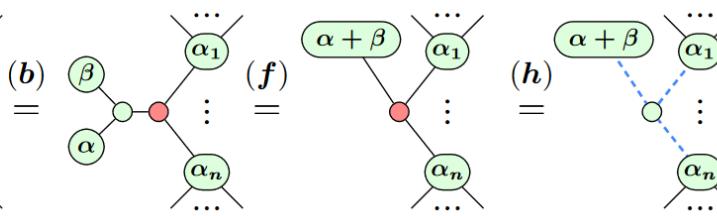
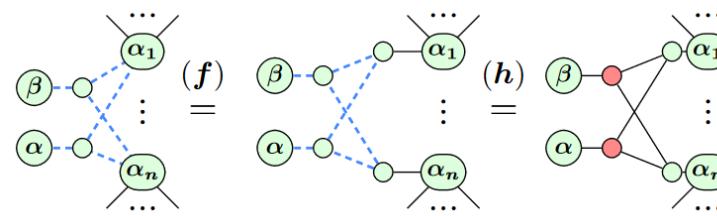
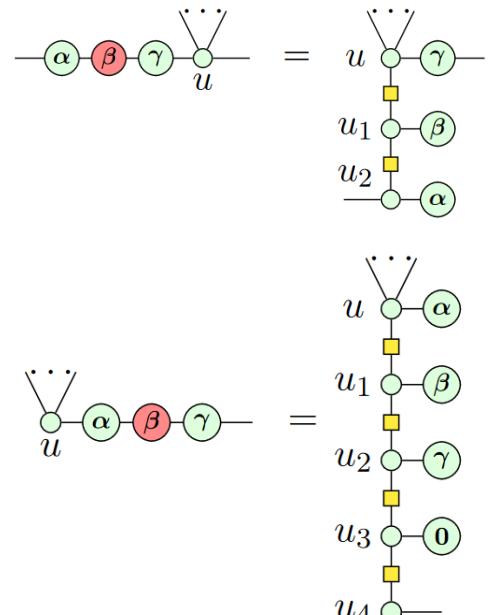
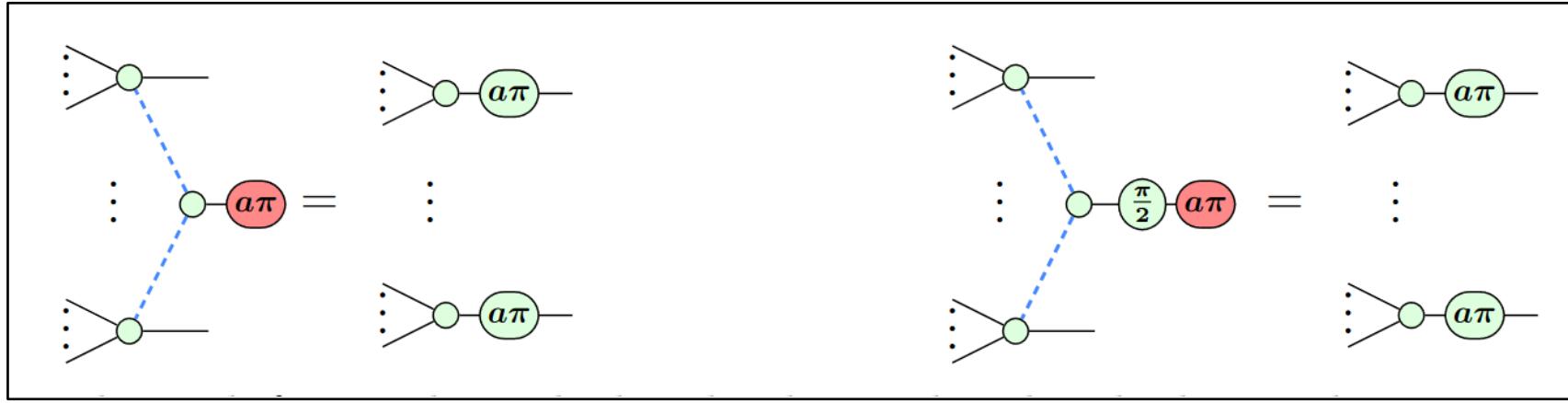
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$$-\left(\frac{\pi}{2}\right) - \alpha = -\left(\alpha + \frac{\pi}{2}\right)$$

$$\pi + \alpha = \alpha + \pi$$

$$\begin{aligned} \text{---} \left(\frac{\pi}{2}\right) \text{---} \alpha &= \text{---} \left(\frac{\pi}{2}\right) \text{---} \left(\frac{\pi}{2}\right) \text{---} \left(\frac{\pi}{2}\right) \text{---} \left(\frac{\pi}{2}\right) \text{---} \alpha - \frac{\pi}{2} = \text{---} \left(\frac{\pi}{2}\right) \text{---} \square \text{---} \alpha - \frac{\pi}{2} = \text{---} \left(\frac{\pi}{2}\right) \text{---} \alpha - \frac{\pi}{2} = \text{---} \left(\frac{\pi}{2}\right) \text{---} \left(\frac{\pi}{2} - \alpha\right) \\ \text{---} \left(\frac{\pi}{2}\right) \text{---} \left(\frac{\pi}{2}\right) \text{---} \alpha &= \text{---} \left(\frac{\pi}{2}\right) \text{---} \left(\frac{\pi}{2}\right) \text{---} \left(\frac{\pi}{2}\right) \text{---} \left(\frac{\pi}{2}\right) \text{---} \alpha = \text{---} \left(\frac{\pi}{2}\right) \text{---} \square \text{---} \alpha = \text{---} \left(\frac{\pi}{2}\right) \text{---} \alpha = \text{---} \alpha - \frac{\pi}{2} \end{aligned}$$

$$\begin{array}{c} \text{Diagram A} \\ \text{Diagram B} \end{array} = \begin{array}{c} \text{Diagram C} \end{array}$$

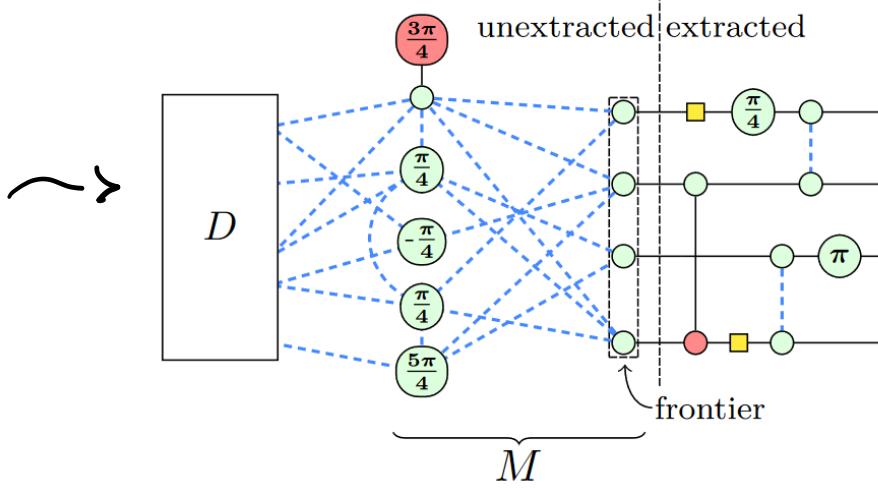
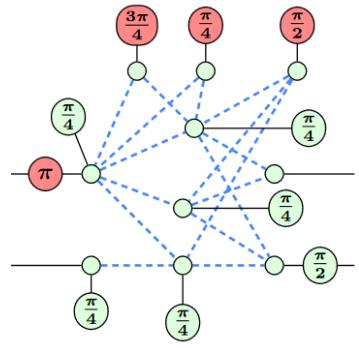
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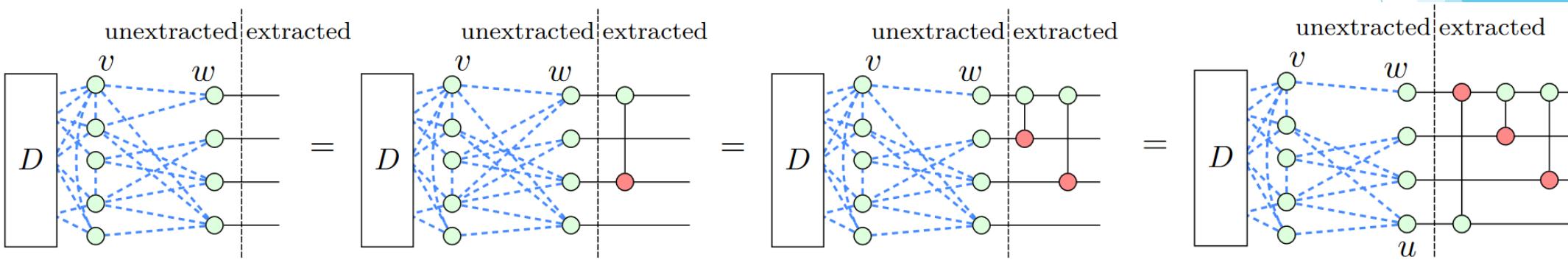
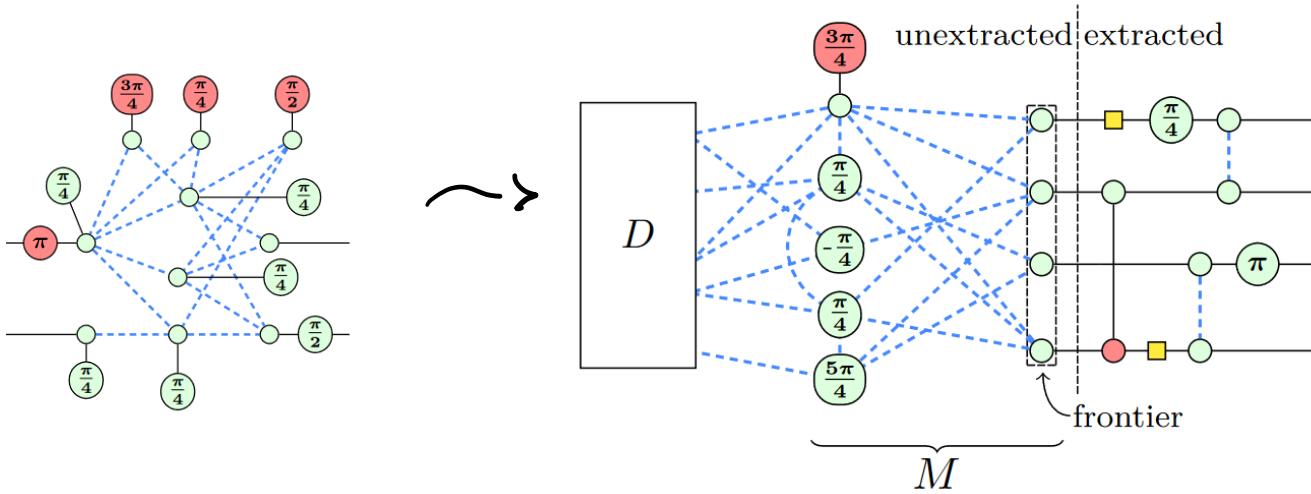
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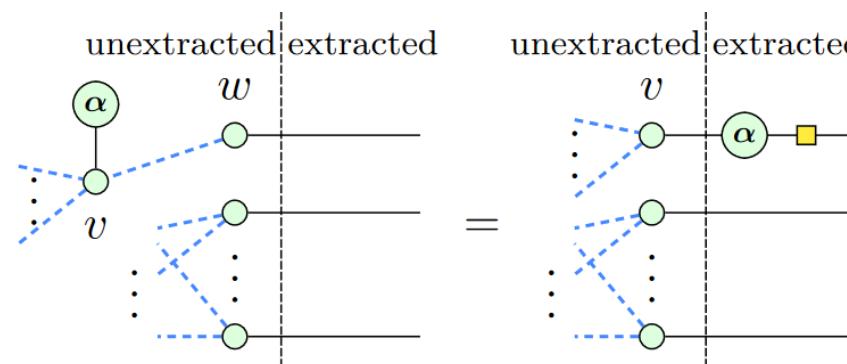
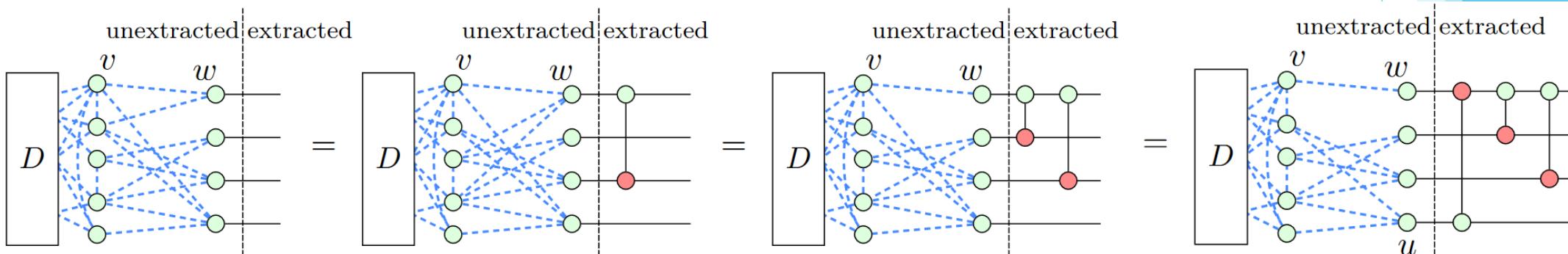
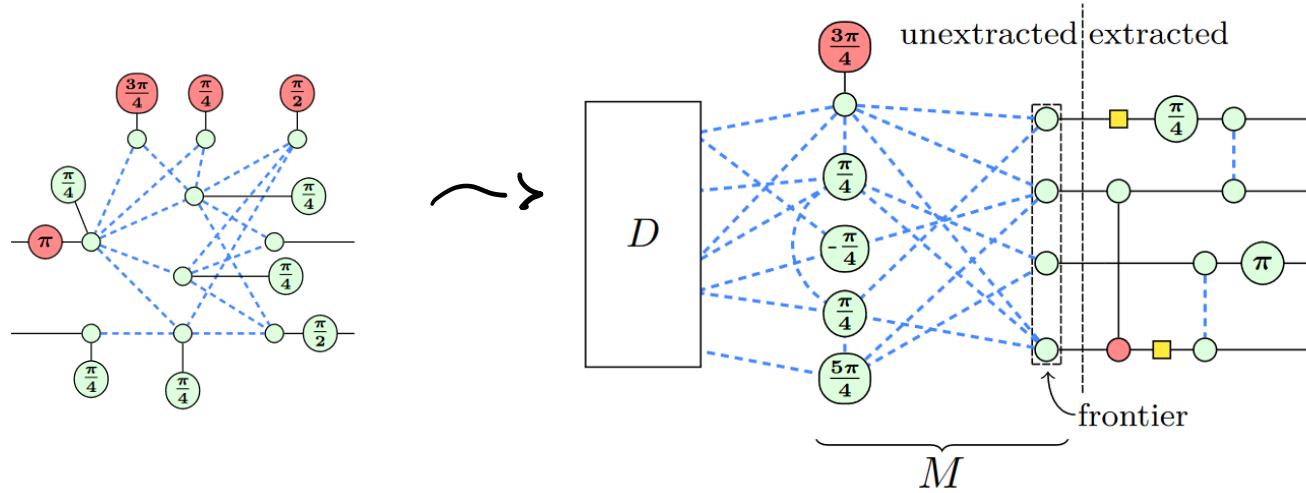
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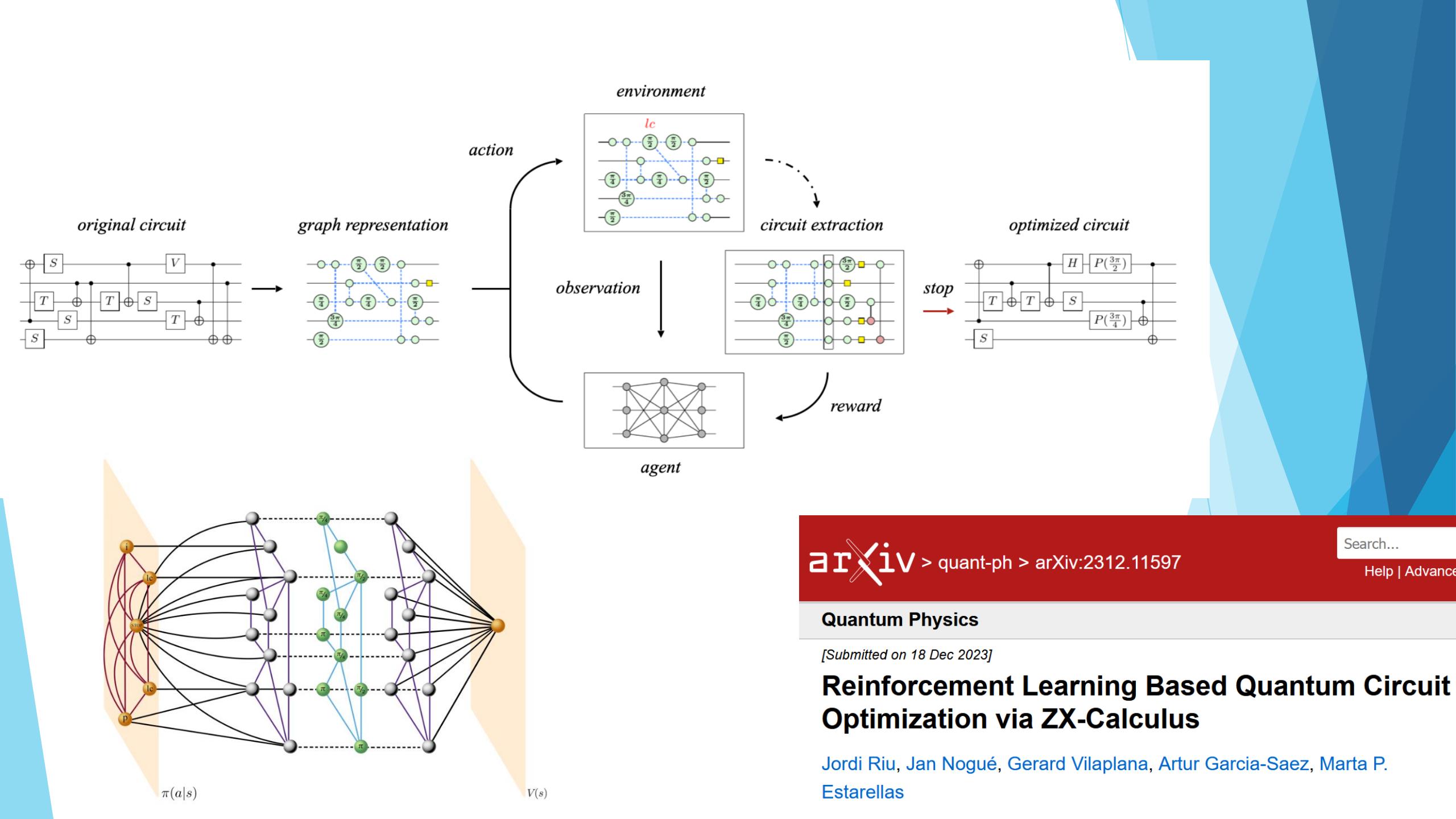
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The ZH and ZX $\&$ Calculi

$$\left[\begin{array}{c} n \\ \vdots \\ a \\ \vdots \\ m \end{array} \right] := |0\rangle^{\otimes n}\langle 0|^{\otimes m} + |1\rangle^{\otimes n}\langle 1|^{\otimes m}$$

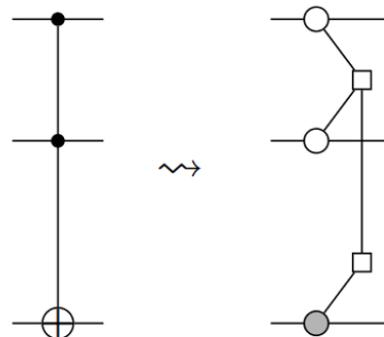
$$\left[\begin{array}{c} n \\ \vdots \\ a \\ \vdots \\ m \end{array} \right] := \sum a^{i_1 \dots i_m j_1 \dots j_n} |j_1 \dots j_n\rangle\langle i_1 \dots i_m|$$

$$\left[\begin{array}{c} n \\ \vdots \\ \alpha \\ \vdots \\ m \end{array} \right] \mapsto \left[\begin{array}{c} n \\ \vdots \\ a \\ \vdots \\ m \end{array} \right] e^{i\alpha}$$

$$\text{H} \mapsto \frac{1}{\sqrt{2}} \begin{pmatrix} & 1 \\ 1 & \end{pmatrix}$$

$$\left[\begin{array}{c} n \\ \vdots \\ \alpha \\ \vdots \\ m \end{array} \right] := \left[\begin{array}{c} n \\ \vdots \\ H \\ \vdots \\ m \end{array} \right] \left[\begin{array}{c} n \\ \vdots \\ H \\ \vdots \\ m \end{array} \right]$$

$$\text{AND} = \dots$$



$$\wedge Z = \dots \rightsquigarrow \wedge^n Z = \dots$$

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Quantum Physics

[Submitted on 6 May 2018 (v1), last revised 29 Jan 2019 (this version, v2)]

ZH: A Complete Graphical Calculus for Quantum Computations Involving Classical Non-linearity

Miriam Backens, Aleks Kissinger

(ZS1)

$$\begin{array}{c} n \\ \backslash \dots / \\ \circlearrowleft \circlearrowright \\ \dots \\ m \end{array} = \begin{array}{c} n \\ \backslash \dots / \\ \circlearrowleft \circlearrowright \\ \dots \\ m \end{array}$$

(HS1)

$$\begin{array}{c} n \\ \backslash \dots / \\ \square \\ \square \\ a \\ \dots \\ m \end{array} = [2] \begin{array}{c} n \\ \backslash \dots / \\ a \\ \dots \\ m \end{array}$$

(ZS2)

$$\begin{array}{c} \circlearrowleft \\ \circlearrowright \\ | \end{array} = \begin{array}{c} \circlearrowleft \\ | \end{array} = \begin{array}{c} | \end{array}$$

(HS2)

$$\begin{array}{c} \square \\ \square \\ | \end{array} = [2] \begin{array}{c} | \end{array}$$

(BA1)

$$\begin{array}{c} n \\ \backslash \dots / \\ \circlearrowleft \circlearrowright \\ \dots \\ m \end{array} = \begin{array}{c} n \\ \backslash \dots / \\ \circlearrowleft \circlearrowright \\ \dots \\ m \end{array}$$

(BA2)

$$\begin{array}{c} n \\ \backslash \dots / \\ \square \\ \dots \\ m \end{array} = \begin{array}{c} n \\ \backslash \dots / \\ \square \square \\ \dots \\ m \end{array}$$

(M)

$$\begin{array}{c} \circlearrowleft \\ a \\ b \\ \circlearrowright \end{array} = \begin{array}{c} ab \end{array}$$

(U)

$$\begin{array}{c} | \\ 1 \end{array} = \begin{array}{c} \circlearrowleft \end{array}$$

(A)

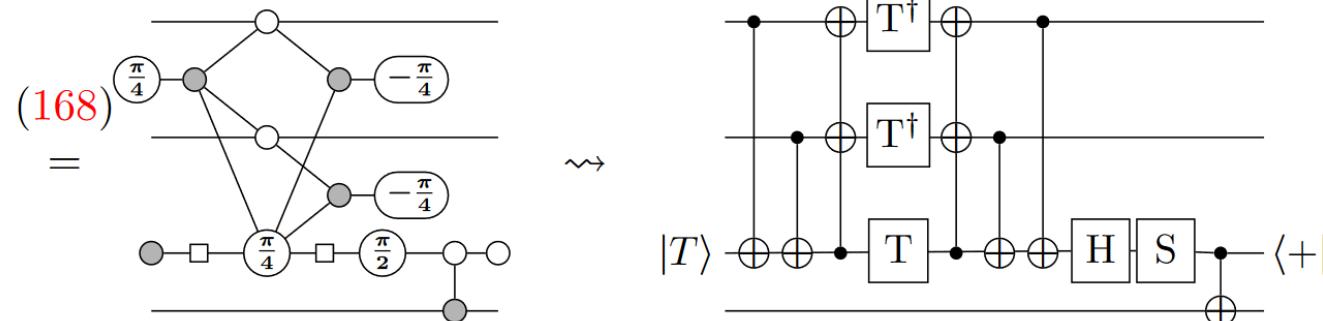
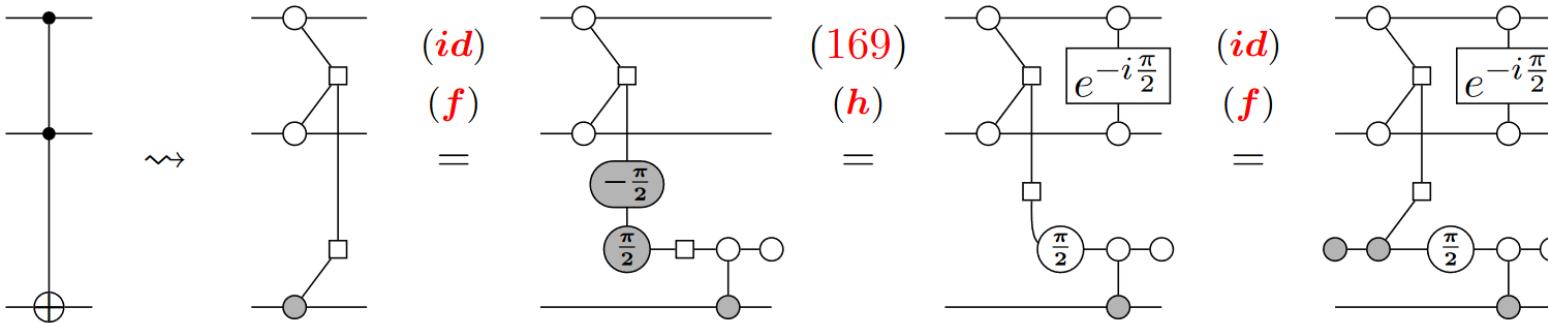
$$\begin{array}{c} \square \\ a \\ b \\ \square \\ \text{\scriptsize \textcircled{1}} \end{array} = [2] \begin{array}{c} | \\ a+b \\ 2 \end{array}$$

(I)

$$\begin{array}{c} | \\ a \\ \circlearrowleft \end{array} = \begin{array}{c} \circlearrowleft \\ a \\ a \\ \square \\ \text{\scriptsize \textcircled{1}} \end{array}$$

(O)

$$[2] \begin{array}{c} \circlearrowleft \\ \square \\ \square \end{array} = \begin{array}{c} \circlearrowleft \\ \square \\ \square \end{array} \begin{array}{c} \circlearrowleft \\ \circlearrowright \\ \circlearrowleft \end{array} \begin{array}{c} \circlearrowright \\ \circlearrowleft \\ \circlearrowright \end{array}$$



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Quantum Physics

[Submitted on 6 May 2018 (v1), last revised 29 Jan 2019 (this version, v2)]

ZH: A Complete Graphical Calculus for Quantum Computations Involving Classical Non-linearity

Miriam Backens, Aleks Kissinger

[ZX&.1]

A diagram showing two spiders, one labeled α and one labeled β , merging into a single spider labeled $\alpha + \beta$.

[ZX&.2]

A diagram showing a complex spider labeled α being simplified into a single spider labeled α .

[ZX&.3]

A diagram showing three adjacent spiders merging into a single spider.

[ZX&.4]

A diagram showing two adjacent spiders merging into a single spider.

[ZX&.5]

A diagram showing a spider with a self-loop merging with another spider.

[ZX&.6]

A diagram showing a spider with a self-loop and a connection to another spider merging with another spider.

[ZX&.7]

A diagram showing a spider with a self-loop and a connection to another spider merging with another spider.

[ZX&.8]

A diagram showing two spiders connected in series merging into a single spider.

[ZX&.9]

A diagram showing a complex spider with multiple inputs and outputs merging into a single spider.

[ZX&.10]

A diagram showing a spider labeled π being simplified.

[ZX&.11]

A diagram showing a spider labeled π being simplified.

[ZX&.12]

A diagram showing a spider labeled π being simplified.

[ZX&.13]

A diagram showing a spider labeled π being simplified.

[ZX&.14]

A diagram showing a spider labeled π being simplified.

[ZX&.15]

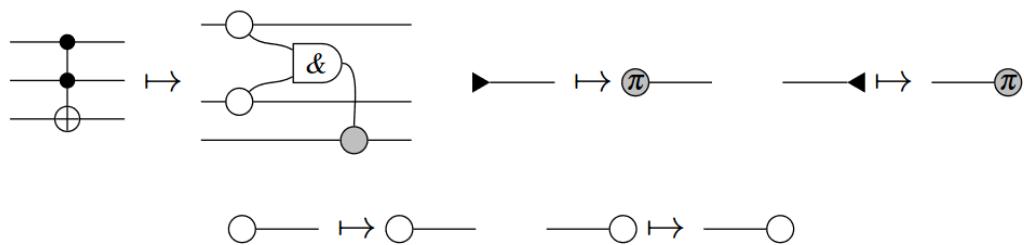
A diagram showing a spider labeled π being simplified.

[ZX&.16]

A diagram showing a spider labeled π being simplified.

[ZX&.17]

A diagram showing a spider labeled π being simplified.



arXiv > cs > arXiv:2004.05287

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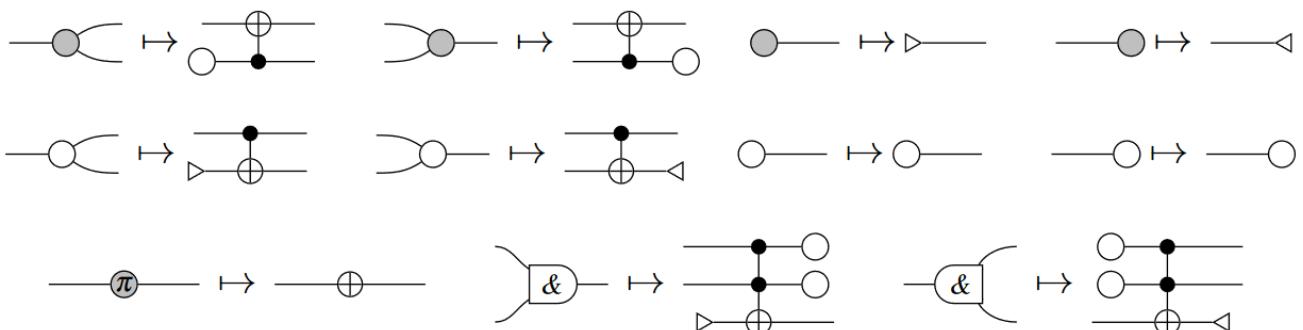
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Computer Science > Logic in Computer Science

[Submitted on 11 Apr 2020 (v1), last revised 6 Sep 2021 (this version, v5)]

The ZX&-calculus: A complete graphical calculus for classical circuits using spiders

Cole Comfort (University of Oxford)



Thank you!